Imp Questions For

BSC (MATHS) - Mathematics

3^{rd} year - V sem - Linear Algebra

1. Define the term SubSpace of a vector space.
2. Show that H is a Sub Space of $\mathbb{R}^3$. i.e $H = \left\{ \begin{bmatrix} s \\ t \\ 0 \end{bmatrix} ; \text{ s and t are real} \right\}$.
3. Given v_1 and v_2 in a vector space V, let $H = \text{Span}\{v_1, v_2\}$. Show That H is a Subspace of V.
4. Show that H is a SubSpace i.e $H = \{(a-3b, b-a, a, b) ; a, b \in \mathbb{R}\}$
5. For what values of h will y be in the Subspace of $\mathbb{R}^3$ spanned by v_1, v_2, v_3 if $v_1 = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}$, $v_2 = \begin{bmatrix} 5 \\ -4 \\ -7 \end{bmatrix}$, $v_3 = \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}$ and $y = \begin{bmatrix} -4 \\ 3 \\ h \end{bmatrix}$
6. Define The terms, i) Null Spaces

   ii) Column Spaces

   iii) Vector Space.
7. Find a Spanning set for The null Space of The matrix

$$A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$$

8. Prove that The Column Space of an $m \times n$ matrix A is a Subspace of $\mathbb{R}^m$
9. Find a matrix A such That $W = \text{Col } A$

$$W = \left\{ \begin{bmatrix} 6a-b \\ a+b \\ -7a \end{bmatrix} : a, b \in \mathbb{R} \right\}$$

10. Define The term linear Transformation of a vector space.

11. With $A = \begin{bmatrix} +2 & 4 & -2 & 1 \\ -2 & -5 & 7 & 3 \\ 3 & 7 & -8 & 6 \end{bmatrix}$

(i) if the column space of A is a subspace of $\mathbb{R}^k$, what is k?

(ii) if the null space of A is a subspace of $\mathbb{R}^k$, what is k?

(iii) find a non zero vector in Col A, in Nul A.

(iv) Let $u = \begin{bmatrix} 3 \\ -2 \\ -1 \\ 0 \end{bmatrix}$ and $v = \begin{bmatrix} 3 \\ -1 \\ 3 \end{bmatrix}$

Determine if u is in Nul A. Could u be in Col A?

Determine if v is in Col A. Could v be in Nul A?

12. Let A be an n×n matrix. If Col A = Nul A, show that Nul $A^2 = \mathbb{R}^n$.

13. Define the terms linearly independent and linearly dependent sets.

14. Let $v_1 = \begin{bmatrix} 3 \\ 0 \\ -6 \end{bmatrix}$, $v_2 = \begin{bmatrix} -4 \\ 1 \\ 7 \end{bmatrix}$, $v_3 = \begin{bmatrix} -2 \\ 1 \\ 5 \end{bmatrix}$. Determine if $\{v_1, v_2, v_3\}$ is a basis for $\mathbb{R}^3$.

15. Let $v_1 = \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$, $v_3 = \begin{bmatrix} 6 \\ 16 \\ -5 \end{bmatrix}$, and $H = \text{Span}\{v_1, v_2, v_3\}$.

Note that $v_3 = 5v_1 + 3v_2$, and show that Span $\{v_1, v_2, v_3\}$ = Span $\{v_1, v_2\}$. Then find a basis for the subspace H.

16. State and prove that spanning set theorem.

17. $A = \begin{bmatrix} 1 & 4 & 0 & 2 & -1 \\ 3 & 12 & 1 & 5 & 5 \\ 2 & 8 & 1 & 3 & 2 \\ 5 & 20 & 2 & 8 & 8 \end{bmatrix}$ Find a basis for Col A.

(i.e pivot columns of a matrix A are form a basis for Col A)

18. Let V and W be vector spaces, let $T: V \to W$ and $U: V \to W$ be linear Transformations, and let $\{v_1, \ldots, v_p\}$ be a basis for V. if $T(v_j) = U(v_j)$ for every value of j between 1 and p, show that $T(x) = U(x)$ for every vector x in V.

19. which ones are L.I and which ones span $\mathbb{R}^3$.

i) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ ii) $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

iii) $\begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ -5 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -2 \end{bmatrix}$

20. Find a basis for the null spaces of the matrix

$$\begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & -5 & 4 \\ 3 & -2 & 1 & -2 \end{bmatrix}, \begin{bmatrix} 1 & 0 & -5 & 1 & 4 \\ -2 & 1 & 6 & -2 & -2 \\ 0 & 2 & -8 & 1 & 9 \end{bmatrix}.$$

21. State unique representation Theorem.

22. Consider a basis $B = \{b_1, b_2\}$ for $\mathbb{R}^2$, where $b_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $b_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. Suppose an x in $\mathbb{R}^2$ has the coordinate vector $[x]_B = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$ find x. [i.e we know $[x]_B = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \Rightarrow x = c_1 v_1 + c_2 v_2$

23. Find x, $B = \left\{ \begin{bmatrix} 3 \\ -5 \end{bmatrix}, \begin{bmatrix} -4 \\ 6 \end{bmatrix} \right\}, [x]_B = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$.

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24. Let $b_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, $b_2 = \begin{bmatrix} 5 \\ -6 \end{bmatrix}$, $x = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$. Find the coordinate vector $[x]_B$ of x relative to the basis
$B = \{b_1, b_2\}$

25. The set $B = \{1+t^2, t+t^2, 1+2t+t^2\}$ is a basis for $\mathbb{P}_2$.
Find the coordinate vector of $p(t) = 1+4t+7t^2$ relative to B
(Hint: Find $[P]_B$ i.e $p(t) = c_1 b_1 + c_2 b_2 + c_3 b_3$ Solve for c_1, c_2, c_3))

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26. Find the dimensions of the null space and the column space of

$$A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -8 \end{bmatrix}, \quad A = \begin{bmatrix} 3 & 4 \\ -6 & 10 \end{bmatrix}$$

27. Let H be an n-dimensional subspace of an n-dimensional vector space V. Show that $H = V$.

28. Find a basis and state the dimension $\left\{ \begin{bmatrix} 4s \\ -3s \\ -t \end{bmatrix} : s, t \in \mathbb{R} \right\}$

29. Find basis for the row space, the column space, & the null space of the matrix

$$A = \begin{bmatrix} -2 & -5 & 8 & 0 & -17 \\ 1 & 3 & -5 & 1 & 5 \\ 3 & 11 & -19 & 7 & 1 \\ 1 & 7 & -13 & 5 & -3 \end{bmatrix}$$

30. State and prove that the Rank Theorem.

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31. Verify that Rank $uv^T \leq 1$ if $u = \begin{bmatrix} 2 \\ -3 \\ 5 \end{bmatrix}$ and $v = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$.

32. Let $u = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. Find v in $\mathbb{R}^3$ such that $\begin{bmatrix} 1 & -3 & 4 \\ 2 & -6 & 8 \end{bmatrix} = uv^T$.

33. Let $b_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$, $b_2 = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$, $c_1 = \begin{bmatrix} -7 \\ 9 \end{bmatrix}$, $c_2 = \begin{bmatrix} -5 \\ 7 \end{bmatrix}$, and

Consider the basis for $\mathbb{R}^2$ given by $B = \{b_1, b_2\}$ and $C = [\{c_1, c_2\}]$. i) Find the change of coordinates matrix from C to B

ii) Find the change of coordinates matrix from B to C.

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34. Let $A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$, $u = \begin{bmatrix} 6 \\ -5 \end{bmatrix}$, and $v = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$. Are

u and v eigen vectors of A.

Show that 7 is an eigen value of matrix A and find the corresponding eigen vectors.

35. Let $A = \begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix}$. An eigen value of A is 2,

Find a basis for the corresponding eigen space.

36. If $v_1, \ldots, v_r$ are eigen vectors that correspond to distinct eigenvalues $\lambda_1, \ldots, \lambda_r$ of an $n \times n$ matrix A, Then the set $\{v_1, \ldots, v_r\}$ is linearly independent.

37. Is $\begin{bmatrix} -1+\sqrt{2} \\ 1 \end{bmatrix}$ an eigen vector of $\begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}$? if so,

find the eigen value.

38. Is $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ an eigen vector of $\begin{bmatrix} 3 & 6 & 7 \\ 3 & 3 & 7 \\ 5 & 6 & 5 \end{bmatrix}$? if so,

find the eigen value.

39. Find the characteristic equation and eigen values of

$A = \begin{bmatrix} 1 & -4 \\ 4 & 2 \end{bmatrix}$, $A = \begin{bmatrix} 7 & -2 \\ 2 & 3 \end{bmatrix}$, $A = \begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix}$.

40. State and prove the Diagonalization Theorem

41. Diagonalize the following matrix $A = \begin{bmatrix} 2 & 4 & 3 \\ -4 & -6 & -3 \\ 3 & 3 & 1 \end{bmatrix}$

42. Compute A^8, where $A = \begin{bmatrix} 4 & -3 \\ 2 & -1 \end{bmatrix}$ & A^4 for $A = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$

43. Suppose $B = \{b_1, b_2\}$ is a basis for V and $C = \{c_1, c_2, c_3\}$ is a basis for W. Let $T : V \longrightarrow W$ be a linear transformation with the property that $T(b_1) = 3c_1 - 3c_2 + 5c_3$ &

$T(b_2) = 4c_1 + 7c_2 - c_3$.

Find the matrix M for T relative to B and C.

(we know $M = \left[[T(b_1)]_c , [T(b_2)]_c \right]$)

44. The mapping $T: \mathbb{P}_2 \to \mathbb{P}_2$ defined by

$$T(a_0 + a_1 t + a_2 t^2) = a_1 + 2a_2 t$$ is a linear transformation

i) find the B-matrix for T, when B is the basis $\{1, t, t^2\}$

ii) verify that $[T(p)]_B = [T]_B [p]_B$ for each p in $\mathbb{P}_2$.

45. Define $T: \mathbb{R}^2 \to \mathbb{R}^2$ by $T(x) = Ax$, where $A = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}$.

Find a basis B for $\mathbb{R}^2$ with the property that the B-matrix for T is a diagonal matrix.

46. Find $T(a_0 + a_1 t + a_2 t^2)$. if T is the linear Transformation from $\mathbb{P}_2$ to $\mathbb{P}_2$ whose matrix relative to $B = \{1, t, t^2\}$ is

$$[T]_B = \begin{bmatrix} 3 & 4 & 0 \\ 0 & 5 & -1 \\ 1 & -2 & 7 \end{bmatrix}$$

Hint for (Ans: $[T(p)]_B = [T]_B [P]_B$)

$\Rightarrow T(p) = c_1 + c_2 t + c_3 t^2$

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47. let $A = \begin{bmatrix} .5 & -.6 \\ .75 & 1.1 \end{bmatrix}$. Find the eigen values of A, and find a basis for each eigen space.

48. Show that if a and b are real, then the eigen values of $A = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ are $a \pm bi$, with corresponding eigen vectors $\begin{bmatrix} 1 \\ -i \end{bmatrix}$ & $\begin{bmatrix} 1 \\ i \end{bmatrix}$.

[Ans: $\lambda = .8 \pm .6i$].

49. Find eigen values $\begin{bmatrix} .1 & .1 \\ -.1 & .1 \end{bmatrix}$

50. $\begin{bmatrix} .8 & -.6 & 0 \\ .6 & .8 & 0 \\ 0 & 0 & 1.07 \end{bmatrix}$

51. Let $v = (1, -2, 2, 0)$. Find a unit vector u in the same direction as v.

52. Let W be the subspace of $\mathbb{R}^2$ spanned by $x = \left(\frac{2}{3}, 1\right)$. Find a unit vector z that is a basis for W.

53. Prove that $W^{\perp}$ is a subspace of $\mathbb{R}^n$.

54. Prove that for let A be an $m \times n$ matrix, the orthogonal complement of the row space of A is the null space of A. and the orthogonal complement of the column space of A is the null space of A^T.

i.e $(\text{Row } A)^{\perp} = \text{nul } A$ and $(\text{Col } A)^{\perp} = \text{Nul } A^T$.

55. Determine $y = \begin{bmatrix} -3 \\ 7 \\ 4 \\ 0 \end{bmatrix}$, $z = \begin{bmatrix} 1 \\ -8 \\ 15 \\ -7 \end{bmatrix}$ is a orthogonal or not.

56. Verify that (parallelogram law for vectors u and v in $\mathbb{R}^n$:

$$\|u+v\|^2 + \|u-v\|^2 = 2\|u\|^2 + 2\|v\|^2.$$

57. Show that $\left\{ u_1 = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}, u_2 = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, u_3 = \begin{bmatrix} -1/2 \\ -2 \\ 7/2 \end{bmatrix} \right\}$ is a orthogonal set.

58. If $S = \{u_1, \dots u_p\}$ is an orthogonal set of non zero vectors in $\mathbb{R}^n$, then S is linearly independent and hence is a basis for the subspace spanned by S.

59. Let $u_1 = \begin{bmatrix} -1/\sqrt{5} \\ 2/\sqrt{5} \end{bmatrix}$ and $u_2 = \begin{bmatrix} 2/\sqrt{5} \\ 1/\sqrt{5} \end{bmatrix}$. Show that $\{u_1, u_2\}$ is a orthogonal basis for $\mathbb{R}^2$.

60. Compute the orthogonal projection of $\begin{bmatrix} 1 \\ 7 \end{bmatrix}$ onto the line through $\begin{bmatrix} -4 \\ 2 \end{bmatrix}$ and the origin.